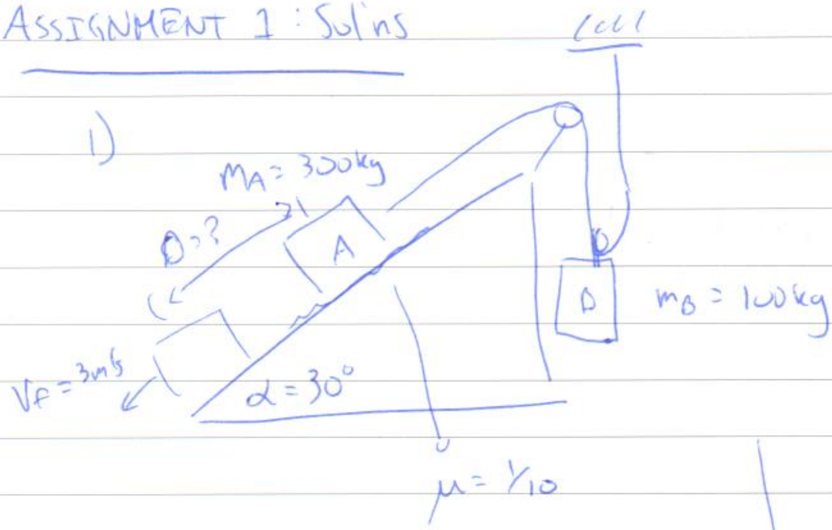
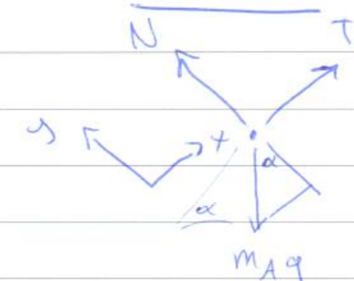
ASSIGNMENT 1: Sol'ns

1)



$\boxed{\text{Find } D.}$

FBD for A:



y: $N - m_A g \cos\alpha = 0$

$$\boxed{\therefore N = m_A g \cos\alpha}$$

Use Cons. of E.

$$\Delta K.E. + \Delta P.E. = \Delta E_{nc}$$

$$\left(\frac{1}{2} m_A V_f^2 + \frac{1}{2} m_B \left(\frac{V_f}{2}\right)^2\right) + \left(-m_A g D \sin\alpha + m_B g \frac{D}{2}\right) = -\mu m_A g \cos\alpha D$$

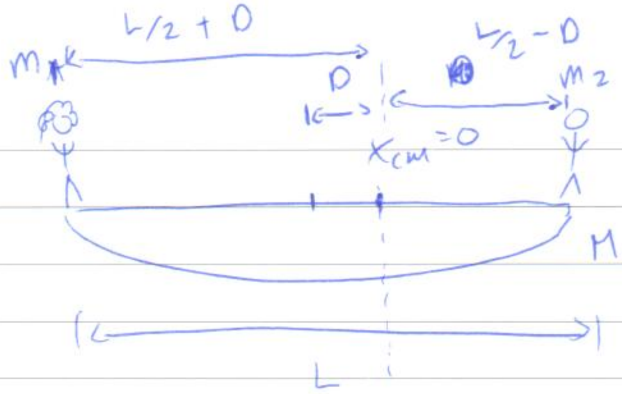
must divide by 2 because of pulley

$$\Rightarrow D\left(-\frac{m_B g}{2} - \mu m_A g \cos\alpha + m_A g \sin\alpha\right) = V_f^2\left(\frac{m_A}{2} + \frac{m_B}{8}\right)$$

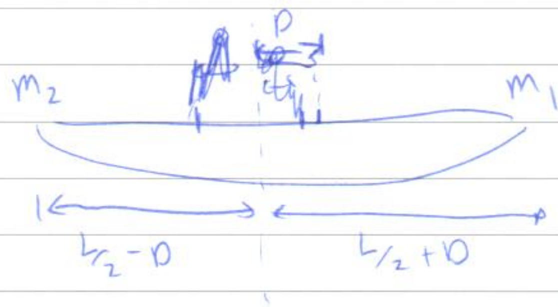
$$\boxed{D = \frac{\frac{V_f^2}{8}\left(4 m_A + m_B\right)}{m_A g \sin\alpha - \frac{m_B g}{2} - \mu m_A g \cos\alpha}}$$

$$= 2.016 \simeq \boxed{2.0 \text{ m}}$$

2) Initial:



Final:



Symmetric! $\therefore$ Displacement = 2D

Find D:

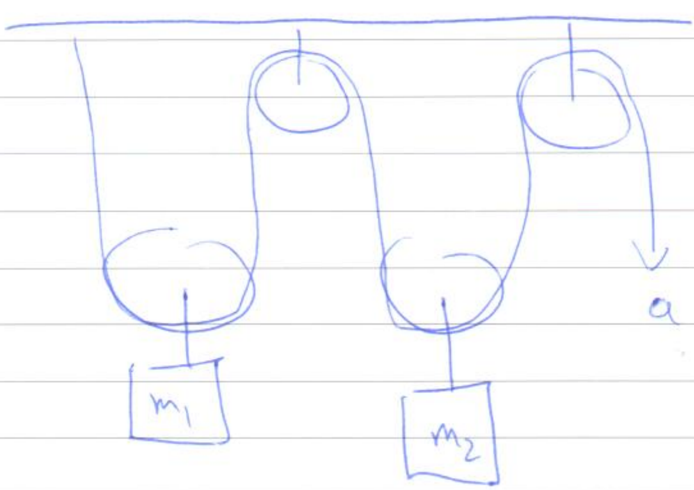
$$X_{cm} = 0 \Rightarrow -m_1(L/2 + D) - MD + m_2(L/2 - D) = 0$$

$$\Rightarrow D(m_2 + m_1 + M) = -\frac{m_1 L}{2} + \frac{m_2 L}{2}$$

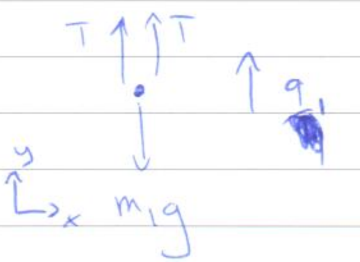
$$\therefore D = \frac{L}{2}\frac{(m_2 - m_1)}{(M + m_1 + m_2)}$$

$$\Rightarrow \boxed{\text{Displacement} = \frac{L(m_2 - m_1)}{M + m_1 + m_2}}$$

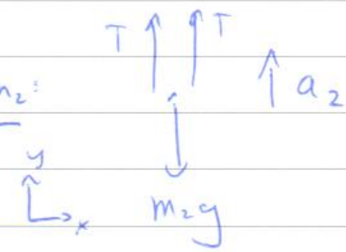
3



FBD m_1:



FBD m_2:



y: $2T - m_1 g = m_1 a_1$ (1) $\quad 2T - m_2 g = m_2 a_2$ (2)

Need a way to relate a_1, a_2 and a.

$$\Rightarrow \boxed{2a_1 + 2a_2 = a} \quad (3)$$

↳ The rope must distribute itself evenly ackross all pulleys!

Eliminat T: $2T \overset{(1)}{=} m_1 a_1 + m_1 g \overset{(2)}{=} m_2 a_2 + m_2 g$

$$\Rightarrow \boxed{m_1 a_1 - m_2 a_2 = g(m_2 - m_1)} \quad (4)$$

Solve (3) & (4) for a_1 and a_2.

$(3) \Rightarrow a_1 = a/2 - a_2 \quad (4) \Rightarrow m_1(a/2 - a_2) - m_2 a_2 = g(m_2 - m_1)$

$$a_2(m_1 + m_2) = m_1\left(\frac{a}{2} + g\right) - m_2 g$$

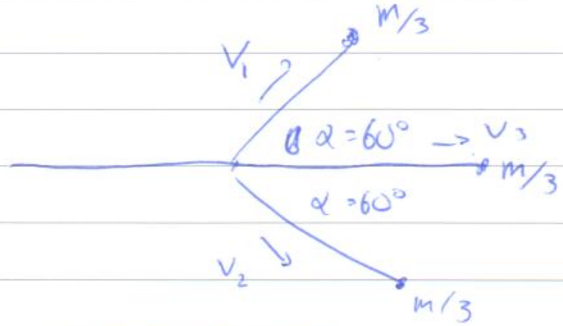
$$\boxed{a_2 = \frac{m_1\left(\frac{a}{2}+g\right) - m_2 g}{m_1+m_2}}$$

$$\boxed{a_1 = \frac{m_2\left(a/2+g\right) - m_1 g}{m_1+m_2}}$$ (by symmetry or by using (3)).

(4) Initial:

m

Final:



By symmetry, $\boxed{v_1 = v_2 \equiv v_f}$ ⇒ or by cons. of p in y-direction

Conservation of $\vec{E}$:

$$\Delta K.E. + \Delta P.E. = \Delta E_{n.c.}$$

$$\frac{1}{2}\frac{m}{3}v_3^2 + 2\cdot\frac{1}{2}\frac{m}{3}v_f^2 - \frac{1}{2}m v_0^2 + 0 = 2\cdot\frac{1}{2}m v_0^2$$

$$\Rightarrow \frac{v_3^2}{3} + \frac{2}{3}v_f^2 - v_0^2 = 2v_0^2$$

$$\boxed{v_3^2 + 2v_f^2 = 9v_0^2} \quad (1)$$

Conservation of $\vec{p}$:

$$x: \quad \boxed{m v_0 = \frac{m}{3}v_3 + \frac{2m}{3}v_f \underbrace{\cos\alpha}_{1/2}}$$

$$\cancel{3v_0 = v_3 + v_f\cos\alpha} \quad \boxed{v_3 = 3v_0 - v_f} \quad (2)$$

(2) => (1): $(3v_0 - v_f)^2 + 2v_f^2 = 9v_0^2$

$\Rightarrow \quad 3v_f^2 = 6v_0 v_f \quad \Rightarrow \boxed{v_f = 2v_0}$ (or 0)
(not valid)

$\therefore$ (2) $\Rightarrow \boxed{v_3 = 3v_0 - 2v_0 = v_0}$