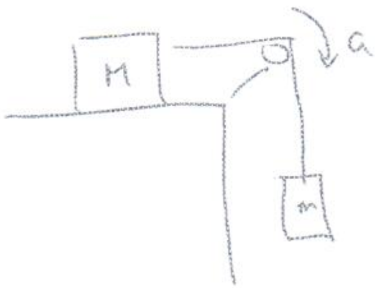
Physical Problem Solving

Exam Solutions

1) Classical Mechanics: Boxes & Pulleys

Method 1 (easy): Conservation of E!

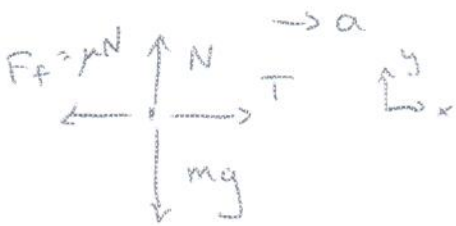


$$\Delta K.E = \tfrac{1}{2} m v_f^2 + \tfrac{1}{2} M v_f^2$$
$$= \tfrac{1}{2} v_f^2 (m+M)$$

$$\Delta P.E = -mgd$$

$$W_f = F_f d = \mu M g d$$

Fbd. of M:



y: $N - Mg = 0 \Rightarrow N = Mg$

$$\boxed{\therefore F_f = \mu M g}$$

$$\Delta K.E + \Delta P.E = -W_f$$

$$\therefore \tfrac{1}{2}(m+M) v_f^2 - mgd = -\mu M g d$$

~~$v_f^2 = 2gd\ \therefore v_f = \sqrt{\frac{2gd(M-\mu M)}{m+M}}$~~

$$\boxed{\therefore v_f = \sqrt{\frac{2gd(m-\mu M)}{m+M}}}$$

Method 2: (hard) Newton's Laws

Fbd. of m:

x: $\boxed{-\mu M g + T = Ma}$ (1)

y: $\boxed{T - mg = ma}$ (2)

(2) ⇒ (1): $-\mu M g + m(g+a) = Ma$

$$\Rightarrow a = \frac{g(m-\mu M)}{M+m}$$

But,

$V_f^2 = \underset{0}{V_i^2} + 2ad$ $\quad \boxed{\therefore V_f = \sqrt{\frac{2gd(m-\mu M)}{M+m}}}$

2) Classical Mechanics: Euler-Lagrange Equations

$$L = \frac{1}{2} m\ell^2 \dot\theta^2 - \left(mg\ell(1-\cos\theta) + \frac{1}{2} k\ell^2(1+\sin^2\theta) \right)$$

(a) $\frac{\partial L}{\partial \theta} = -mg\ell\sin\theta - k\ell^2 \sin\theta\cos\theta$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) = \frac{d}{dt}\left(m\ell^2\dot\theta\right) = m\ell^2\ddot\theta$$

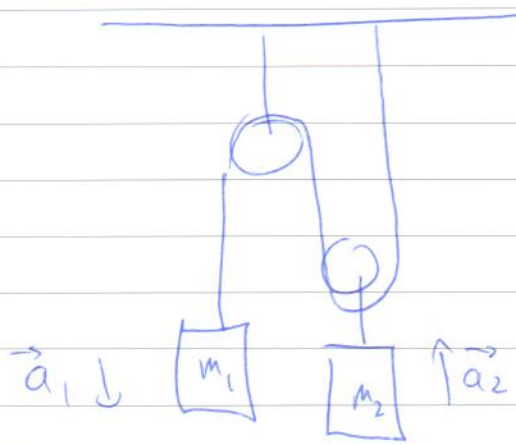
$\Rightarrow \frac{\partial L}{\partial \theta} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) \underset{(1)}{\Rightarrow} \boxed{m\ell^2\ddot\theta = -(mg\ell\sin\theta + k\ell^2\sin\theta\cos\theta)}$

(b) for $\theta \ll 1$ $\sin\theta \to \theta$; $\sin\theta\cos\theta \to \theta$

(1) ⇒ $\boxed{\therefore \ddot\theta \approx -\left(\frac{mg+k\ell}{m\ell}\right)\theta}$

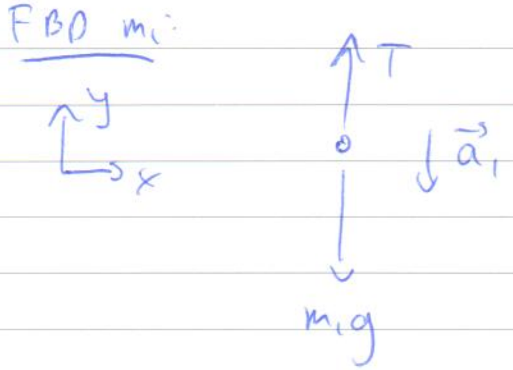
(c) $\ddot\theta = -\omega^2\theta \Rightarrow \omega^2 = \frac{mg+k\ell}{m\ell}$ or $\boxed{\omega = \sqrt{\frac{mg+k\ell}{m\ell}}}$

2) Classical Mechanics II : Pulley Problem



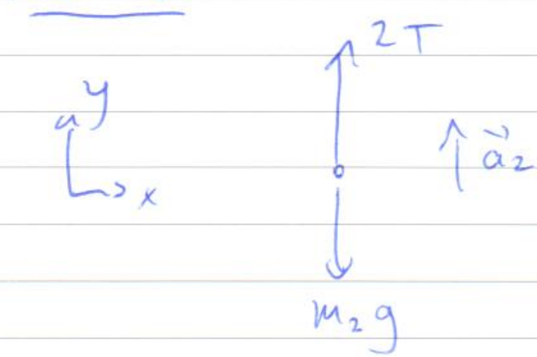
a) Assume the directions above for $\vec{a}_1$ and $\vec{a}_2$.

FBD m_1:



$$y: \boxed{T - m_1 g = -m_1 a_1} \quad (1)$$

FBD m_2:



$$y: \boxed{2T - m_2 g = m_2 a_2} \quad (2)$$

Note that the conservation of the length of the rope implies

$$\boxed{|\vec{a}_1| = 2|\vec{a}_2| \equiv 2a}$$

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In terms of 'a', we have

(1) & (2) $\Rightarrow$
$$\begin{cases} T = m_1(g-2a) & (1) \\ T = \frac{m_2}{2}(g+a) & (2) \end{cases}$$

$$\Rightarrow \therefore m_1(g-2a) = \frac{m_2}{2}(g+a)$$

$$\Rightarrow a\left(\frac{m_2}{2} + 2m_1\right) = g\left(-\frac{m_2}{2} + m_1\right)$$

$$\boxed{\therefore a = g\left(\frac{2m_1 - m_2}{4m_1 + m_2}\right) = \vec{a}_2} \quad (3) \quad \boxed{\therefore |\vec{a}_1| = g\left(\frac{4m_1 - 2m_2}{4m_1 + m_2}\right)}$$

(b) $a = 0 \overset{(3)}{\Rightarrow} \boxed{2m_1 = m_2}$

(c) Block 1 will rise when $a < 0$. This occurs via (3) when

$$2m_1 - m_2 < 0 \Rightarrow \boxed{m_1 < \tfrac{1}{2}m_2}$$

Block 2 will fall in this case. (See arrows in diagram.)

(d) Block 2 will rise when $a > 0$. This occurs via (3) when

$$2m_1 - m_2 > 0 \Rightarrow \boxed{m_1 > \tfrac{1}{2}m_2}$$

Block 1 will fall in this case.

~~(d) Yes Because (e.g.)~~

~~i) ω increases with g (as expected).~~

~~ii) " " " k "~~

~~iii) m and ℓ reduce ω "~~

~~iv) As $k \to 0$; $\omega \to \sqrt{\frac{g}{\ell}}$ $\Rightarrow$ reproduces the simple pendulum.~~

3) Foundations of Mechanics:

(a) 1) Principle of Sufficient Reason

i) Every phenomenon that exist should have a sufficient reason (or "cause") for doing so.

ii) Absolute Space violates this because it exists without cause.

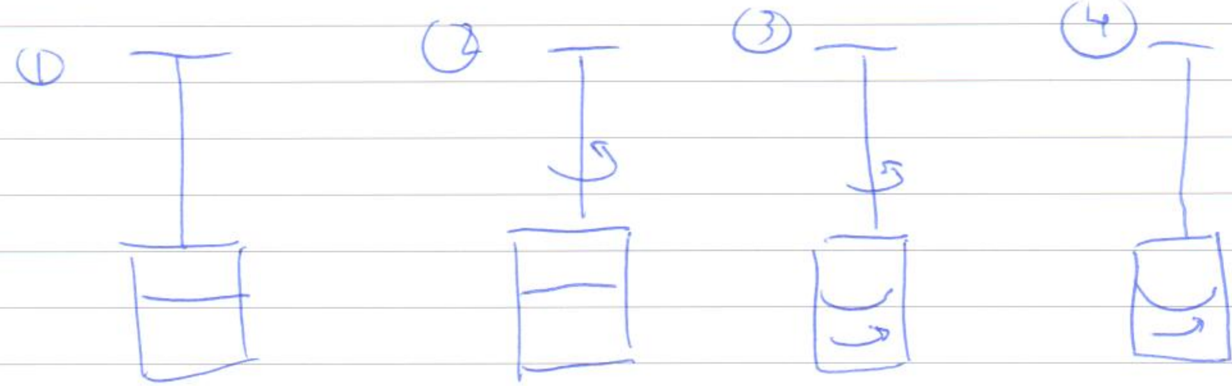
2) Principle of Identity of Indiscernables

i) Two things that share the same property are the same.

ii) No two points on $\mathbb{R}^2$ are distinguishable because they can be removed by translation. Therefore, they are part of the same thing.

(b) i) ii) No! Their explanation violates (2) because the aether has no cause (or effect). It is invisible.

③ ⓒ Newton argued that the outcome of the bucket experiment, particularly the physical effect of the shape of the water, was caused, not by the ~~motion~~ of relative motion of the bucket with respect to the water, but the motion of the water with respect to absolute space.



If you consider the 4 configurations above, you see that the relative motion is uncorrelated with the shape of the water. Only the absolute motion with respect to absolute space is important. See the table below:

Configuration	Abs motion	Rel motion	Shape of water
①	no	no	Flat
②	no	yes	Flat
③	yes	no	Curved
④	yes	yes	Curved.

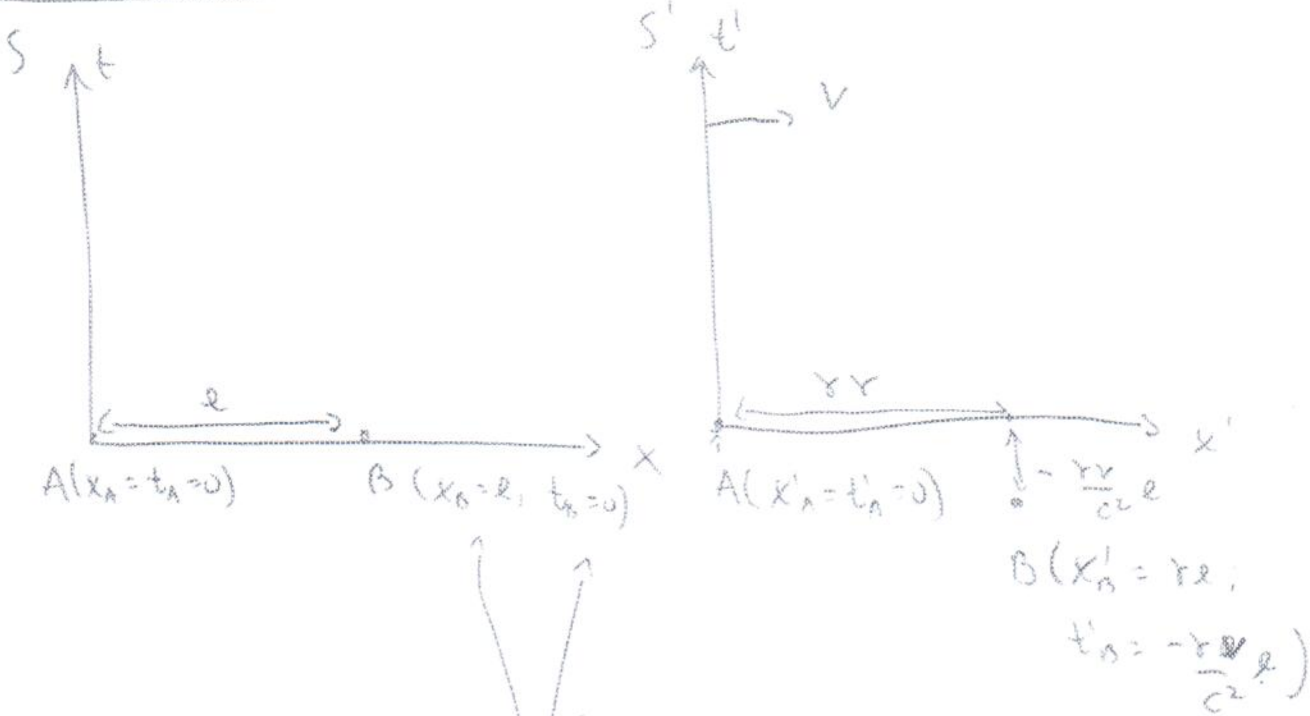
<u>The rule:</u> If (Abs motion = no) then (Shape = Flat)
Else (shape = curved)

works to explain the data. There is no correlation between (Rel motion) and (Shape).

4(6)

4) Relative Simultaneity:

(a)



(b) $x'_A = 0$; $t'_A = 0$ (A is the origin of both)

$$\boxed{x'_B = \gamma(x_B - v t_B) = \gamma\ell}$$ (not necessary)

$$\boxed{t'_B = \gamma\left(t_B - \frac{v}{c^2}x_B\right) = -\frac{\gamma v}{c^2}\ell}$$

$$\boxed{\therefore\ t_B - t'_B = -\frac{\gamma v}{c^2}\ell}$$

B occurs <u>first</u> by $\frac{\gamma v}{c^2}\ell$ seconds.